WEEKLY TEST TARGET JEE – 01 SOLUTION MATHEMATICS 14 JULY 2019

61. (b) $(x+a)^n+(x-a)^n=2\,[x^n+{}^nC_2x^{n-2}a^2+{}^nC_4x^{n-4}a^4+$
${}^nC_6x^{n-6}a^6+.......]$

Here, $n=6, x=\sqrt{2}, a=1$; ${}^6C_2=15, {}^6C_4=15, {}^6C_6=1$

$\therefore (\sqrt{2}+1)^6(\sqrt{2}-1)^6=2[(\sqrt{2})^6+15.(\sqrt{2})^4.1$
$+15(\sqrt{2})^2.1+1.1]$

$=2[8+15\times 4+15\times 2+1]=198$

62. (a) As given $(1+ax)^n=1+8x+24x^2+....$

$\Rightarrow 1+\frac{n}{1}ax+\frac{n(n-1)}{1.2}a^2x^2+....=1+8x+24x^2+....$

$\Rightarrow\ na=8, \frac{n(n-1)}{1.2}a^2=24 \Rightarrow na(n-1)a=48$

$\Rightarrow 8(8-a)=48 \Rightarrow\ 8-a=6 \Rightarrow a=2 \Rightarrow n=4$.

63. (b) We have $(1+x^2)^5(1+x)^4$

$=({}^5C_0+{}^5C_1x^2+{}^5C_2x^4+...)\,({}^4C_0+{}^4C_1x+{}^4C_2x^2+{}^4C_3x^3+{}^4C_4x^4)$

So coefficient of x^5 in $[(1+x^2)^5(1+x)^4]$

$=\ {}^5C_2.{}^4C_1+{}^4C_3.{}^5C_1=60.$

64. (b) $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{2\left[1-\frac{x}{4}\right]^{1/2}}$

$$=\frac{\left[1+\frac{1}{2}(-3x)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(-3x)^2+....\right]+\left[1+\frac{5}{3}(-x)+\frac{5}{3}\frac{2}{3}\frac{1}{2}(-x)^2+....\right]}{2\left[1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}\left(-\frac{x}{4}\right)^2+....\right]}$$

$$=\frac{\left[1-\frac{19}{12}x+\frac{53}{144}x^2-....\right]}{\left[1-\frac{x}{2}-\frac{1}{8}x^2-....\right]}=1-\frac{35}{24}x+....$$

Neglecting higher powers of x, then

$a+bx=1-\frac{35}{24}x \Rightarrow a=1, b=-\frac{35}{24}$.

65. (a) $T_3={}^5C_2.x^2(x^{\log_{10}x})^3=10^6$

Put ${}^5C_2=10$ $\quad[\because \log_{10}10=1]$.

If $x=10$,then $10^3.10^{2.1}=10^5$ is satisfied.

Hence $x=10$.

66. (c) Since $(n+2)^{th}$ term is the middle term in the expansion of $(1+x)^{2n+2}$, therefore $p={}^{2n+2}C_{n+1}$.

Since $(n+1)^{th}$ and $(n+2)^{th}$ terms are middle terms in the expansion of $(1+x)^{2n+1}$, therefore $q={}^{2n+1}C_n$ and $r={}^{2n+1}C_{n+1}$ But ${}^{2n+1}C_n+{}^{2n+1}C_{n+1}={}^{2n+2}C_{n+1}$

$\therefore\ q+r=p$

67. (b) $(x-1)(x-2)(x-3)....(x-100)$

Number of terms = 100;

$\therefore$ Coefficient of x^{99} = $(x-1)(x-2)(x-3)...(x-100)$

$=(-1-2-3-......-100)=-(1+2+.....+100)$

$=-\frac{100\times 101}{2}=-5050.$

68. (a) $T_{r+1}={}^{200}C_r(1)^{200-r}.(x)^r$

Hence coefficient of $x^{100}={}^{200}C_{100}=\binom{200}{100}$.

69. (a) In the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$, the general term is $T_{r+1}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r={}^{11}C_r a^{11-r}\frac{1}{b^r}x^{22-3r}$

For x^7, we must have $22-3r=7\Rightarrow r=5$, and the coefficient of $x^7={}^{11}C_5.a^{11-5}\frac{1}{b^5}={}^{11}C_5\frac{a^6}{b^5}$

Similarly, in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, the general term is $T_{r+1}={}^{11}C_r(-1)^r\frac{a^{11-r}}{b^r}.x^{11-3r}$

For x^{-7} we must have, $11-3r=-7\Rightarrow r=6$, and the coefficient of x^{-7} is ${}^{11}C_6\frac{a^5}{b^6}={}^{11}C_5\frac{a^5}{b^6}$.

As given, ${}^{11}C_5\frac{a^6}{b^5}={}^{11}C_5\frac{a^5}{b^6}\Rightarrow ab=1$.

70. (c) $T_{r+1}={}^5C_r(x^2)^{5-r}\left(\frac{k}{x}\right)^r$

For coefficient of x, $10-2r-r=1 \Rightarrow r=3$

Hence, $T_{3+1}={}^5C_3(x^2)^{5-3}\left(\frac{k}{x}\right)^3$

According to question, $10k^3=270\Rightarrow k=3$.

71. (a) Let the coefficient of three consecutive terms *i.e.* $(r+1)^{th}$, $(r+2)^{th}$, $(r+3)^{th}$ in expansion of $(1+x)^n$ are 165,330 and 462 respectively then, coefficient of $(r+1)^{th}$ term $={}^nC_r=165$

Coefficient of $(r+2)^{th}$ term $={}^nC_{r+1}=330$ and

Coefficient of $(r+3)^{th}$ term $={}^nC_{r+2}=462$

$\therefore \frac{{}^nC_{r+1}}{{}^nC_r}=\frac{n-r}{r+1}=2$

or $n-r=2(r+1)$ or $r=\frac{1}{3}(n-2)$

and $\frac{{}^nC_{r+2}}{{}^nC_{r+1}}=\frac{n-r-1}{r+2}=\frac{231}{165}$

or $165(n-r-1)=231(r+2)$ or $165n-627=396r$

or $165n-627=396\times\frac{1}{3}\times(n-2)$

or $165n-627=132(n-2)$ or $n=11$.

72. (d) ${}^{18}C_{2r+3}={}^{18}C_{r-3}\Rightarrow 2r+3+r-3=18\Rightarrow r=6$

73. (d) Middle term of $(1+x)^{2n}$ is $T_{n+1}={}^{2n}C_nx^n$

$=\frac{(2n)!}{n!\,n!}x^n=\frac{1.3.5....(2n-1)}{n!}2^nx^n$.

74. (b) $(1-x)^{30}={}^{30}C_0x^0-{}^{30}C_1x^1+{}^{30}C_2x^2$

$+......+(-1)^{30}\ {}^{30}C_{30}x^{30}$ ....(i)

$(x+1)^{30}={}^{30}C_0x^{30}+{}^{30}C_1x^{29}+{}^{30}C_2x^{28}$

$+......+{}^{30}C_{10}x^{20}+....+{}^{30}C_{30}x^0$ ....(ii)

Multiplying (i) and (ii) and equating the coefficient of x^{20} on both sides, we get required sum = coefficient of x^{20} in $(1-x^2)^{30}={}^{30}C_{10}$.

75. (c) $(1+3x+3x^2+x^3)^6=\{(1+x)^3\}^6=(1+x)^{18}$

Hence the middle term is 10^{th}.

76. (c) Middle term of $\left(x-\frac{1}{x}\right)^{11}$ is $T_6={}^{11}C_5(x)^6\left(-\frac{1}{x}\right)^5=-462x$

and $T_7={}^{11}C_6(x)^5\left(-\frac{1}{x}\right)^6=\frac{462}{x}$

77. (d) $(9-r)\left(-\frac{1}{6}\right)+r\left(\frac{1}{3}\right)=0\Rightarrow r=3$

So the term independent of y

$={}^9C_3(y^{-1/6})^6(-y^{1/3})^3=-84$.

78. (c) The general term in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is $T_{r+1}={}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)x^{18-3r}$ ......(i)

Now, the coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ ......(ii)

= Sum of the coefficient of the terms x^0, x^{-1} and x^{-3} in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

For x^0 in (i) above, $18-3r=0\Rightarrow r=6$. For x^{-1} in (i) above, there exists no value of r and hence no such term exists. For x^{-3} in (i), $18-3r=-3\Rightarrow r=7$

$\therefore$ For term independent of x, in (ii) the coefficient

$=1\times{}^9C_6(-1)^6\left(\frac{3}{2}\right)^{9-6}\left(\frac{1}{3}\right)^6+2\times{}^9C_7(-1)^7\left(\frac{3}{2}\right)^{9-7}\left(\frac{1}{3}\right)^7$

$=\frac{9.8.7}{1.2.3}.\frac{3^3}{2^3}.\frac{1}{3^6}+2\frac{9.8}{1.2}(-1)\frac{3^2}{2^2}.\frac{1}{3^7}=\frac{7}{18}-\frac{2}{27}=\frac{17}{54}$.

79. (b) Accordingly, $(\sqrt{x})^{10-r}\left(\frac{1}{x^2}\right)^r=x^0\Rightarrow r=2$

Hence the term is ${}^{10}C_2\left(\frac{1}{\sqrt{3}}\right)^8.(\sqrt{3})^2=\frac{5}{3}$.

80. (a) $T_{r+1}={}^{18}C_r(\sqrt{x})^{18-r}\left(-\frac{2}{x}\right)^r={}^{18}C_r x^{9-r/2-r}(-2)^r$

If T_{r+1} is independent of x, then $9-\frac{r}{2}-r=0\Rightarrow r=6$.

So term independent of $x=T_7={}^{18}C_6 2^6$

81. (c) $3^{50}\left(1+\frac{2x}{3}\right)^{50}$

$\therefore \frac{T_{r+1}}{T_r}\ge 1\Rightarrow 102-2r\ge 15r\Rightarrow r\le 6$

82. (b) We know that

$\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+....+n.\frac{C_n}{C_{n-1}}=\frac{n(n+1)}{2}$

Putting $n=15$, then $\frac{15\times(15+1)}{2}=120$.

83. (c) $(1+x)^n={}^nC_0+x.{}^nC_1+x^2.{}^nC_2+....+x^n.{}^nC_n$

Put $x=2$

$\Rightarrow 3^n={}^nC_0+2.{}^nC_1+2^2.{}^nC_2+2^3.{}^nC_3+....+2^n.{}^nC_n$.

84. (d) We have $C_0^2-2C_1^2+3C_2^2-.....+(-1)^n(n+1)C_n^2$

$=[C_0^2-C_1^2+C_2^2-....+(-1)^nC_n^2]-$

$[C_1^2-2C_2^2+3C_3^2....-(-1)^n nC_n^2]$

$=(-1)^{n/2}\frac{n!}{(n/2)!(n/2)!}-(-1)^{(n/2)-1}.\frac{1}{2}n\ {}^nC_{n/2}$

$=(-1)^{n/2}.\frac{n!}{(n/2)!(n/2)!}.\left(1+\frac{n}{2}\right)$

Therefore the value of the given expression is

$\frac{2(n/2)!(n/2)!}{n!}\times(-1)^{n/2}.\frac{(n)!}{(n/2)!(n/2)!}\left(1+\frac{n}{2}\right)$

$=(-1)^{n/2}(2+n)$

85. (c) $(x+a)^n={}^nC_0x^n+{}^nC_1x^{n-1}a+{}^nC_2x^{n-2}a^2+{}^nC_3x^{n-3}a^3+.....$

But by the condition,

$A={}^nC_0x^n+{}^nC_2x^{n-2}a^2+{}^nC_4x^{n-4}a^4+......$

and $B={}^nC_1x^{n-1}a+{}^nC_3x^{n-3}a^3+......$

Hence $AB = \frac{1}{4}\left\{(x+a)^{2n} - (x-a)^{2n}\right\}$

or $\quad 4AB = (x+a)^{2n} - (x-a)^{2n}$

86. (b) $(x+a)^n = x^n + {}^nC_1 x^{n-1}a + {}^nC_2 x^{n-2}a^2 + {}^nC_3 x^{n-3}a^3 + ...$

$= (x^n + {}^nC_2 x^{n-2}a^2 + ...$

$+ ({}^nC_1 x^{n-1}a + {}^nC_3 x^{n-3}a^3 + ...)$

$= P + Q$

$\therefore\ (x-a)^n = P - Q$, As the terms are alter. $+ve$ and $-ve$

$\therefore\ P^2 - Q^2 = (P+Q)(P-Q) = (x+a)^n (x-a)^n$

$P^2 - Q^2 = (x^2 - a^2)^n$

87. (c) Putting $x = 1$ in $(1 + x - 3x^2)^{2163}$.

we get sum of the coefficients as $(1+1-3)^{2163} = (-1)^{2163} = -1$.

88. (b) We have a = sum of the coefficient in the expansion of $(1 - 3x + 10x^2)^n = (1 - 3 + 10)^n = (8)^n$

$\Rightarrow (1 - 3x + 10x^2)^n = (2)^{3n}$, [Putting $x = 1$] .

Now, b = sum of the coefficients in the expansion of $(1 + x^2)^n = (1+1)^n = 2^n$. Clearly, $a = b^3$

89. (b) By hypothesis, $2^n = 4096 = 2^{12} \Rightarrow n = 12$

Since n is even, hence greatest coefficient

$= {}^nC_{n/2} = {}^{12}C_6 = \frac{12.11.10.9.8.7}{1.2.3.4.5.6} = 924$.

90. (b) Accordingly, $(\alpha - 2 + 1)^{35} = (1 - \alpha)^{35}$

$\Rightarrow (\alpha - 1)^{35} = (1 - \alpha)^{35} \Rightarrow \alpha = 1$